**Table 5.2:** Reduction factors for carbon steel for the design at elevated temperatures

| Steel Temperature $\theta_a$ | Reduction factors at temperature $\theta_a$ relative to the value of $f_y$ or $E_a$ at 20°C | | | |
|---|---|---|---|---|
| | Reduction factor (relative to $f_y$) for effective yield strength $k_{y,\theta}=f_{y,\theta}/f_y$ | Reduction factor (relative to $f_y$) for proportional limit $k_{p,\theta}=f_{p,\theta}/f_y$ | Reduction factor (relative to $E_a$) for the slope of the linear elastic range $k_{E,\theta}=E_{a,\theta}/E_a$ | Reduction factor (relative to $f_y$) for the design strength of hot rolled and welded thin walled sections (Class 4) $k_{0.2p,\theta}=f_{0.2p,\theta}/f_y$ |
| 20 °C | 1.000 | 1.000 | 1.000 | 1.000 |
| 100 °C | 1.000 | 1.000 | 1.000 | 1.000 |
| 200 °C | 1.000 | 0.807 | 0.900 | 0.890 |
| 300 °C | 1.000 | 0.613 | 0.800 | 0.780 |
| 400 °C | 1.000 | 0.420 | 0.700 | 0.650 |
| 500 °C | 0.780 | 0.360 | 0.600 | 0.530 |
| 600 °C | 0.470 | 0.180 | 0.310 | 0.300 |
| 700 °C | 0.230 | 0.075 | 0.130 | 0.130 |
| 800 °C | 0.110 | 0.050 | 0.090 | 0.070 |
| 900 °C | 0.060 | 0.0375 | 0.0675 | 0.050 |
| 1000 °C | 0.040 | 0.0250 | 0.0450 | 0.030 |
| 1100 °C | 0.020 | 0.0125 | 0.0225 | 0.020 |
| 1200 °C | 0.000 | 0.0000 | 0.0000 | 0.000 |
| NOTE: For intermediate values of the steel temperature, linear interpolation may be used. | | | | |



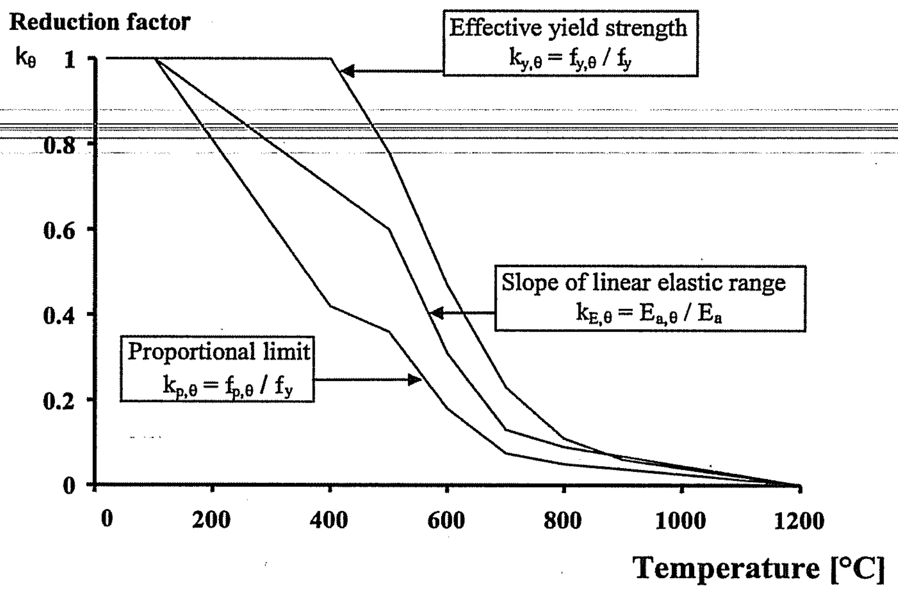


**Fig. 5.6:** Reduction factors for the stress-strain relationship of carbon steel at elevated temperatures (see Fig. 3.2 from EN 1993-1-2)

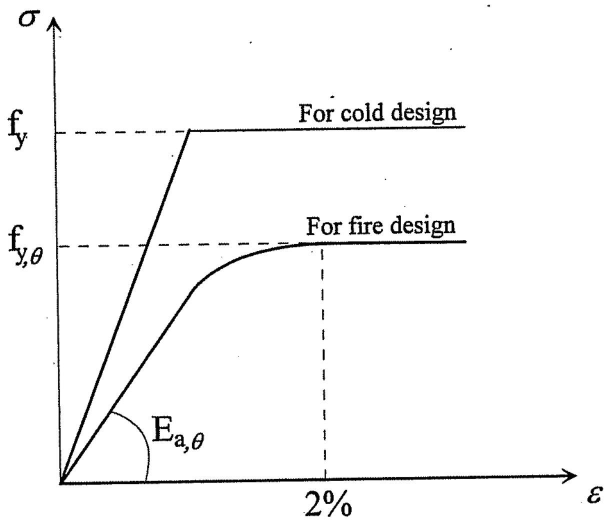


**Fig. 5.15:** Stress-strain relationship

### 5.4.2. Tension members

The design value of the tension force in the fire situation, $N_{fi,Ed}$, at each cross section should satisfy the following condition:

$$\frac{N_{fi,Ed}}{N_{fi,\theta,Rd}} \leq 1.0 \tag{5.22}$$

where the design resistance $N_{fi,\theta,Rd}$ of a tension member with a uniform temperature $\theta_a$ should be determined from:

$$N_{fi,\theta,Rd} = k_{y,\theta} N_{Rd} [\gamma_{M0} / \gamma_{M,fi}] \tag{5.23}$$



where:

- $k_{y,\theta}$ is the reduction factor for the yield strength of steel at uniform temperature $\theta_a$, reached at time $t$, see Section 5.2;
- $\gamma_{M0}$ is the partial safety factor for the resistance of cross sections, whatever the class is;
- $\gamma_{M,fi}$ is the partial safety factor for the fire situation;
- $N_{Rd}$ is the design resistance of the cross section $N_{pl,Rd}$ for normal temperature design, according to EN 1993-1-1, and given by:

$$N_{pl,Rd} = \frac{A f_y}{\gamma_{M0}} \tag{5.24}$$

The recommended value for $\gamma_{M0}$ and $\gamma_{M,fi}$ is 1.0, but different values may be defined in the National Annex.

Substituting Eq. (5.23) in to Eq. (5.22) leads to

$$N_{fi,\theta,Rd} = Ak_{y,\theta}f_y / \gamma_{M,fi} \qquad (5.25)$$

According to Annex D of Part 1.2 of Eurocode 3, net-section failure at fastener holes does not need to be considered, provided that there is a fastener in each hole. This is because the steel temperature is lower at joints due to the presence of additional material (e.g. bolts, fittings, etc.).

The design resistance $N_{fi,\theta,Rd}$ at time $t$ of a tension member with a non-uniform temperature distribution across the cross section may be determined from:

$$N_{fi,\theta,Rd} = \sum_{i=1}^{n} A_i k_{y,\theta,i} f_y / \gamma_{M,fi} \qquad (5.26)$$

where the subscript $i$ refers to an elemental area of the cross section in which the temperature is considered as uniform.

The design resistance $N_{fi,t,Rd}$ at time $t$ of a tension member with a non-uniform temperature distribution may conservatively be taken as equal to the design resistance $N_{fi,\theta,Rd}$ of a tension member with a uniform steel temperature $\theta_a$ equal to the maximum steel temperature $\theta_{a,\max}$ reached at time $t$.

### 5.4.3. Compression members



The design value of the compression force in the fire situation, $N_{b,fi,Ed}$, at each cross section should satisfy the following condition:

$$\frac{N_{b,fi,Ed}}{N_{b,fi,t,Rd}} \leq 1.0 \qquad (5.27)$$

where the design buckling resistance $N_{b,fi,t,Rd}$ at time $t$ of a compression member with a Class 1, Class 2 or Class 3 cross section with a uniform temperature $\theta_a$ should be determined from:

$$N_{b,fi,t,Rd} = \chi_{fi} Ak_{y,\theta}f_y / \gamma_{M,fi} \qquad (5.28)$$

where

$k_{y,\theta}$ is the reduction factor for the yield strength of steel at uniform temperature $\theta_a$ ,reached at time $t$, see Section 5.2;

$\chi_{fi}$ is the reduction factor for flexural buckling in the fire design situation, given by Eq. (5.29).

The value of $\chi_{fi}$ should be taken as the lower of the values of $\chi_{y,fi}$ and $\chi_{z,fi}$ determined according to:

$$\chi_{fi} = \frac{1}{\phi_\theta + \sqrt{\phi_\theta^2 - \bar{\lambda}_\theta^2}} \tag{5.29}$$

where

$$\phi_\theta = \frac{1}{2}\left[1 + \alpha\bar{\lambda}_\theta + \bar{\lambda}_\theta^2\right] \tag{5.30}$$

and the imperfection factor, $\alpha$, proposed by Franssen *et al.*, 2005 is given by

$$\alpha = 0.65\sqrt{235 / f_y} \tag{5.31}$$

The non-dimensional slenderness $\bar{\lambda}_\theta$ for the temperature $\theta_a$, is given by

$$\bar{\lambda}_\theta = \bar{\lambda}\sqrt{k_{y,\theta} / k_{E,\theta}} \tag{5.32}$$

where

$\bar{\lambda}$ - is the non-dimensional slenderness at room temperature given by Eq. (5.33) using the buckling length in fire situation $l_{fi}$ (see Fig. 5.16).



The non-dimensional slenderness at room temperature, $\bar{\lambda}$, is given by

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} \tag{5.33}$$

where $\lambda$ is the member slenderness, evaluated with the buckling length in fire situation, $l_{fi}$, and given by

$$\lambda = \frac{l_{fi}}{i} \tag{5.34}$$

where $i$ is the radius of gyration of the cross section and $\lambda_1$ is given by (EN 1993-1-1)

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93.9\varepsilon \tag{5.35}$$

with

$$\varepsilon = \sqrt{\frac{235}{f_y}} \quad (f_y \text{ in N/mm}^2) \tag{5.36}$$

where

$E$ - is the Young's modulus at room temperature;
$f_y$ - is the yield strength at room temperature.

The buckling length $l_{fi}$ of a column for the fire design situation should generally be determined as for normal temperature design. In the case of a braced frame, the buckling length $l_{fi}$ of a continuous column may be determined by considering it as fixed to the fire compartments above and below, provided that the fire resistance of the building components that separate these fire compartments is not less than the fire resistance of the column.

For example, in the case of a braced frame in which each storey comprises a separate fire compartment with sufficient fire resistance, in an intermediate storey the buckling length $l_{fi}$ of a continuous column may be taken as $l_{fi} = 0.5L$ and in the top storey the buckling length may be taken as $l_{fi} = 0.7L$, where $L$ is the system length in the relevant storey, see Fig. 5.16.



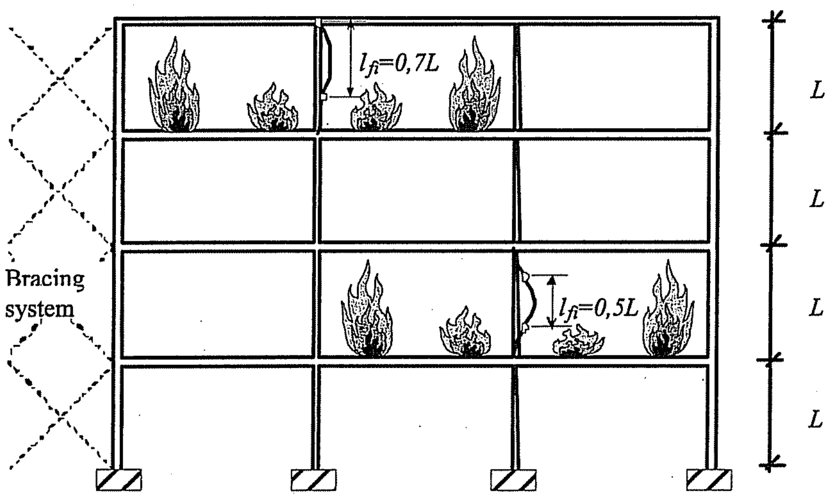


**Fig. 5.16:** Buckling lengths $l_{fi}$ of columns in braced frames

When designing using nominal fire exposure, the design resistance $N_{b,fi,t,Rd}$ at time $t$, of a compression member with a non-uniform temperature distribution, may be taken as equal to the design resistance $N_{b,fi,\theta,Rd}$ of a compression member, with a uniform steel temperature $\theta_a$ equal to the maximum steel temperature $\theta_{a,\max}$ reached at time $t$.

As the non-dimensional slenderness in the fire situation $\bar{\lambda}_\theta$ depends on the temperature, an iterative procedure is needed if the critical temperature corresponding to a given applied load is to be evaluated, for verification in the time or in the temperature domain (see Eq. 5.1a and Eq. 5.1c). Convergence is usually very fast and one or two iterations are normally sufficient if Eq. (5.32) is approximated, for the first iteration, using the same approximation used in Eq. (5.18), (i.e., $\sqrt{k_{E,\theta}/k_{y,\theta}} \approx 0.85$). This gives the following normalised slenderness at high temperatures, Franssen *et al.* (2009).

$$\bar{\lambda}_\theta = \bar{\lambda}\sqrt{k_{y,\theta} / k_{E,\theta}} \approx \bar{\lambda} / 0.85 \approx 1.2\bar{\lambda} \qquad (5.37)$$

Example 5.3 illustrates this procedure.

### 5.4.4. Shear resistance

The design value of the shear force in a fire situation, $V_{fi,Ed}$ at each cross section should satisfy



$$\frac{V_{fi,Ed}}{V_{fi,t,Rd}} \leq 1.0 \qquad (5.38)$$

where the design shear resistance $V_{fi,t,Rd}$ at time $t$ for a Class 1, Class 2 or Class 3 cross section should be determined from:

$$V_{fi,t,Rd} = k_{y,\theta,web} V_{Rd} [\gamma_{M0} / \gamma_{M,fi}] \qquad (5.39)$$

where:

$V_{Rd}$ - is the shear resistance of the gross cross section for normal temperature design, according to EN 1993-1-1, and given in Eq. (5.40);

$\theta_{web}$ - is the average temperature of the web. ;

$k_{y,\theta,web}$ - is the reduction factor for the yield strength of steel at the web temperature $\theta_{web}$ .

$$V_{fi,Ed} = \frac{q_{fi,Ed} \cdot l}{2} = 67.6 \text{ kN}$$

The shear area is

$$A_v = A - 2bt_f + (t_w + 2r)t_f$$
$$= 5380 - 2 \cdot 150 \cdot 10.7 + (7.1 + 2 \cdot 15)10.7 = 2567 \text{ mm}^2$$

Considering that the reduction factor for the yield strength for 624 ºC is equal to the degree of utilisation

$$k_{y,\theta} = \mu_0 = 0.388$$

and the design value of the shear force at that temperature is

$$V_{fi,t,Rd} = \frac{A_v k_{y,\theta} f_y}{\sqrt{3}\gamma_{M0}} = \frac{2567 \cdot 0.388 \cdot 235}{\sqrt{3}} \times 10^{-3} = 135 \text{ kN} > N_{fi,Ed} \quad \text{OK !}$$

If interpolation in Table 5.2 was made for 624 ºC, $k_{y,\theta} = 0.412$ and

$$V_{fi,t,Rd} = \frac{A_v k_{y,\theta} f_y}{\sqrt{3}\gamma_{M0}} = \frac{2567 \cdot 0.412 \cdot 235}{\sqrt{3}} \times 10^{-3} = 144 \text{ kN}$$



**Example 5.3: Unprotected column under axial compression**

Consider a 3.5 m long HE 180 B column in S275 grade steel, located in an intermediate storey of a braced frame and subject to a compression load of $N_{fi,Ed} = 495$ kN in the fire situation. Assuming that the column doesn't have any fire protection and that the required fire resistance is R30, verify the fire resistance in each of the following domains:

a) Temperature;
b) Time;
c) Resistance.

**Solution:**

**Classification of the cross section:**

The relevant geometrical characteristics of the profile for the cross section classification are

$h = 180$ mm
$b = 180$ mm
$t_w = 8.5$ mm
$t_f = 14$ mm
$r = 15$ mm
$c = b/2 - t_w/2 - r = 70.75$ mm (flange)
$c = h - 2t_f - 2r = 122$ mm (web)

As the steel grade is S275

$$\varepsilon = 0.85\sqrt{235/f_y} = 0.786$$

The class of the flange in compression is

$$c/t_f = 70.75/14 = 5.1 < 9\varepsilon = 7.07 \Rightarrow \text{Class 1}$$

The class of the web in compression is

$$d/t_w = 122/8.5 = 14.4 < 33\varepsilon = 25.9 \Rightarrow \text{Class 1}$$

The cross section of the HE 180 B in fire situation is Class 1. This classification could be directly obtained using the table for cross-sectional classification of Annex F, Vila Real *et al.* (2009b).

**Evaluation of the critical temperature:**

For the HE 180 B:

Area, $A = 6525\ \text{mm}^2$

Minimum radius of gyration, $i = 45.7$ mm

The design value of the compression load in fire situation: $N_{fi,Ed} = 495$ kN

The buckling length for intermediate storey is: $l_{fi} = 0.5L = 0.5 \times 3.5 = 1.75$ m.

The non-dimensional slenderness at elevated temperature is given by Eq. (5.32)

$$\overline{\lambda}_\theta = \overline{\lambda} \cdot \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} = 1.00$$

This is temperature dependent and an iterative procedure is needed to calculate the critical temperature. Starting with a temperature of 20 °C at which $k_{y,\theta} = k_{E,\theta} = 1.0$, equations (5.33), (5.34) and (5.35) give:

$$\bar{\lambda}_\theta = \bar{\lambda} = \frac{\lambda}{\lambda_1} = \frac{l_{fi}/i}{93.9\sqrt{235/275}} = \frac{0.5 \cdot 350/4.57}{93.9\sqrt{235/275}} = 0.441$$

The reduction factor for flexural buckling $\chi$ is evaluated using Eq. (5.29):

$$\alpha = 0.65\sqrt{235/f_y} = 0.65\sqrt{235/275} = 0.601$$

and

$$\phi = \frac{1}{2}(1 + 0.601 \cdot 0.441 + 0.441^2) = 0.730$$

Therefore the reduction factor for flexural buckling is:

$$\chi = \frac{1}{0.730 + \sqrt{0.730^2 - 0.441^2}} = 0.763$$

The design value of the buckling resistance $N_{b,fi,t,Rd}$ at time $t = 0$, is obtained from Eq. (5.28):

$$N_{b,fi,0,Rd} = \chi_{fi} A f_y / \gamma_{M,fi} = 1368\,\text{kN}$$



and the degree of utilisation takes the value:

$$\mu_0 = \frac{N_{fi,Ed}}{N_{fi,0,Rd}} = \frac{495}{1368} = 0.362$$

For this degree of utilisation Eq. (5.85) gives a critical temperature, $\theta_{a,cr} = 635\,°\text{C}$. Using this temperature, the non-dimensional slenderness $\bar{\lambda}_\theta$ can be corrected, which leads to another critical temperature. The iterative procedure should continue until convergence is reached, as illustrated in the next table:

| $\theta$ [°C] | $\sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}}$ | $\overline{\lambda}_\theta = \overline{\lambda} \cdot \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}}$ | $\chi_{fi}$ | $N_{fi,0,Rd} = \chi_{fi} A f_y$ [kN] | $\mu_0 = \frac{N_{fi,Ed}}{N_{fi,0,Rd}}$ | $\theta_{a,cr}$ [°C] |
|---|---|---|---|---|---|---|
| 20 | 1.00 | 0.441 | 0.763 | 1368 | 0.362 | 635 |
| 635 | 1.25 | 0.552 | 0.703 | 1262 | 0.392 | 623 |
| 623 | 1.24 | 0.548 | 0.705 | 1265 | 0.391 | 623 |

After three iterations a critical temperature of $\theta_{a,cr} = 623$ °C is obtained.

If at the first iteration the non-dimensional slenderness was approximated by Eq. (5.37), then

$$\overline{\lambda}_\theta = 1.2\overline{\lambda} = 1.2 \times 0.441 = 0.529$$

and the iteration sequence should be:

| $\theta$ [°C] | $\sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}}$ | $\overline{\lambda}_\theta = \overline{\lambda} \cdot \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}}$ | $\chi_{fi}$ | $N_{fi,0,Rd} = \chi_{fi} A f_y$ [kN] | $\mu_0 = \frac{N_{fi,Ed}}{N_{fi,0,Rd}}$ | $\theta_{a,cr}$ [°C] |
|---|---|---|---|---|---|---|
| $\theta$ (?) | 1.20 | 0.529 | 0.715 | 1283 | 0.385 | 625 |
| 625 | 1.24 | 0.549 | 0.705 | 1265 | 0.391 | 623 |
| 623 | 1.24 | 0.548 | 0.705 | 1265 | 0.391 | 623 |

Using the approximated value of the non-dimensional slenderness to start the iterative procedure, the value of the critical temperature at the second iteration differs by only 2 °C from the value obtained at the first iteration, and the iterative process could be stopped.

The verification of the fire resistance of the column may be now made.

a) The section factor of the HE 180 B is $A_m / V = 159$ m$^{-1}$.

The box value for the section factor $[A_m / V]_b$ takes the value

$$[A_m / V]_b = \frac{2\times(b+h)}{A} = \frac{2\times(0.18+0.18)}{65.25\times10^{-4}} = 110.3 \ \text{m}^{-1}$$

and the shadow factor $k_{sh}$

$$k_{sh} = 0.9[A_m / V]_{box} / [A_m / V] = 0.9 \cdot 110.3 / 159 = 0.624$$

The modified section factor is:

$$k_{sh}[A_m / V] = 0.624 \cdot 159 = 99.2 \ \text{m}^{-1}$$

This value could be directly obtaining from the table of Annex E, Vila Real *et al.*, 2009a.

Interpolating Table A.5 yields the following temperature after 30 minutes:

$$\theta_d = 766 \ ^\circ\text{C}$$

and

$$\theta_d > \theta_{a,cr} \qquad \Rightarrow \qquad \text{not satisfactory.}$$

b) By double interpolation of Table A.5 the time needed to reach a temperature of 623 °C is

$$t_{fi,d} = 17.4 \ \text{min}$$



and

$$t_{fi,d} < t_{fi,requ} \qquad \Rightarrow \qquad \text{not satisfactory.}$$

c) The reduction factors for the yield strength and the Young's modulus after 30 minutes of fire exposure are, interpolating in Table 5.2 for a temperature of 766 °C:

$$k_{y,\theta} = 0.1508 \text{ and } k_{E,\theta} = 0.1036$$

The design value of the buckling resistance is obtained from

$$N_{b,fi,t,Rd} = \chi_{fi} A k_{y,\theta} f_y / \gamma_{M,fi}$$

The non-dimensional slenderness at 766 °C, is

$$\overline{\lambda}_\theta = \overline{\lambda} \cdot \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} = 0.441\sqrt{\frac{0.1508}{0.1036}} = 0.532$$

and using

$$\phi_\theta = \frac{1}{2}\left[1 + \alpha\overline{\lambda}_\theta + \overline{\lambda}_\theta^{\,2}\right]$$

with

$$\alpha = 0.65\sqrt{235 / f_y}$$

gives

$$\phi_\theta = 0.8014$$

and the reduction factor for the flexural buckling is:

$$\chi_{fi} = \frac{1}{\phi_\theta + \sqrt{\phi_\theta^{\,2} - \overline{\lambda}_\theta^{\,2}}} = 0.714$$

The design value of the buckling resistance after 30 minutes of fire exposure, takes the value:

$$N_{b,fi,t,Rd} = \chi_{fi} A k_{y,\theta} f_y / \gamma_{M,fi} = 0.714 \cdot 6525 \cdot 0.1508 \cdot 275 \times 10^{-3} / 1.0 = 193\,\text{kN}$$

and

$$N_{b,fi,t,Rd} < N_{fi,d} \qquad \Rightarrow \qquad \text{not satisfactory.}$$

The column does not fulfil the required fire resistance R30.

**Example 5.4: Protected Column under axial compression**

Consider a 2.8 m long HE 220 B column in S235 grade steel located in an intermediate storey of a building subject to a permanent load of G = 730 kN and a variable load of Q = 500 kN. If the column is fire protected by 20 mm thick gypsum boards evaluate the fire resistance time of the columns assuming it heated on all four sides.