LTBEAM

REPORT ON VALIDATION TESTS

1. INTRODUCTION

In order to validate the calculation modules of LTBEAM, comparisons between LTBEAM results and results obtained from other software have been undertaken.

Up to now, the following actions have been carried out :

- Comparison tests LTBEAM/ANSYS
- Comparison tests LTBEAM/DRILL
- Comparison with tests results from literature
- Comparison LTBEAM/FINELG for tapered beams

2. COMPARISON TESTS LTBEAM/ANSYS

2.1 Presentation

In this section are presented results of validation tests of **LTBEAM** by comparison with results obtained from F.E. simulations with **ANSYS V5.6** carried out at CTICM *(many thanks to Pierre-Olivier MARTIN*).

Steel properties considered in calculations are : E = 210000 MPa and ν = 0,3.

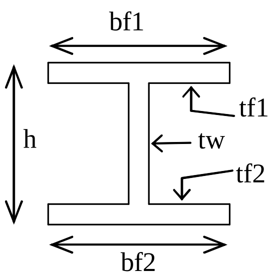
Tests are classified in the following sets :

- ✓ Set 40 : Use of "Formatted data"
- ✓ Set 50 : Beams with intermediate support(s)
- ✓ Set 60 : Cantilever beams
- ✓ Set 70 : Beams under uniformly distributed load
- ✓ Set 80 : Beam with intermediate lateral restraint(s)
- ✓ Set 100 : T Beams
- ✓ Set 110 : Boundary restraint conditions.

For a best overview of the 65 tests performed, a summary table of tests parameters is given hereafter.

For each test, the following information is given in the hereafter tables :

- ✓ the test number
- ✓ the dimensions of the section :



h : Height : h
tf1 : Upper flange width : bf1 and thickness
tf2 : Lower flange width : bf2 and thickness
tw : Web thickness

(no radius at flange-to-web junctions)

- ✓ the LTB restraints conditions and the loading
- ✓ the results of the calculations :
  - o the critical load factor calculated by LTBeam : μ_{LTB},
  - o the critical load factor calculated by ANSYS : μ_{ANS},
  - o the difference between results, calculated with the formula : $\Delta = \frac{|\mu_{ANS} - \mu_{LTB}|}{\mu_{ANS}}$

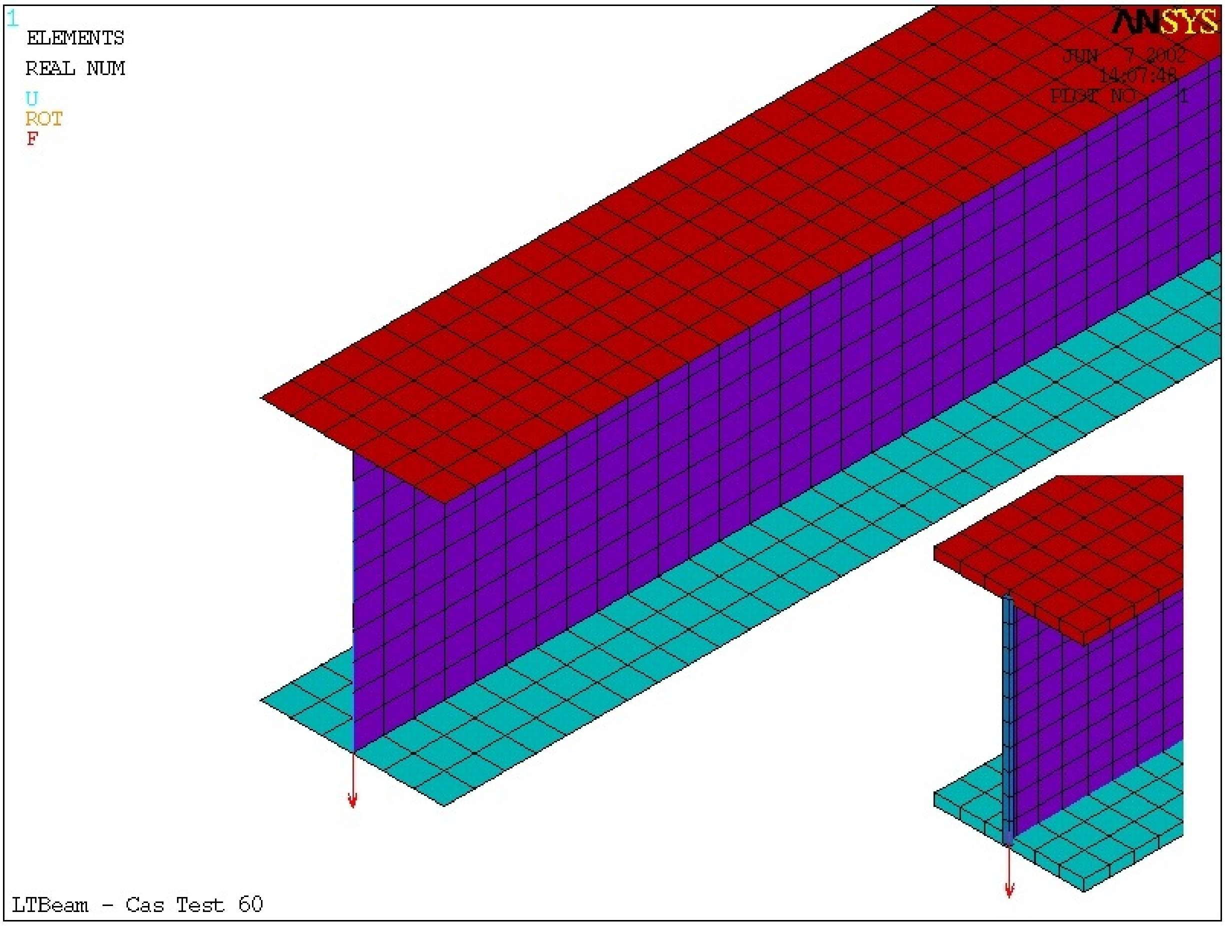
Some information about numerical simulations with ANSYS

For all tests, finite elements **SHELL 63** are used for modelling and are defined in the mid-plane of walls.

A appropriate density of meshing is used. For example, 6923 nodes and 6610 elements are defined for Test 60. The figure hereafter gives an idea of the meshing used for this example and most of the others. This figure also shows how beam elements (BEAM 4) are used at support locations and point load locations in order to stiffen the cross-section and to avoid local buckling effects.

ANSYS performs an **Eigenvalue Buckling Analysis** to find out the critical load factor μ_{ANS} .

All ANSYS data files have been stored and are available.



2.2 Overview table of tests

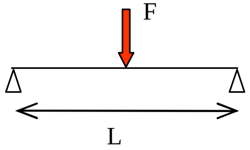
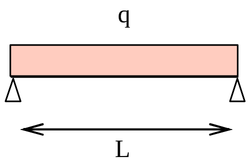
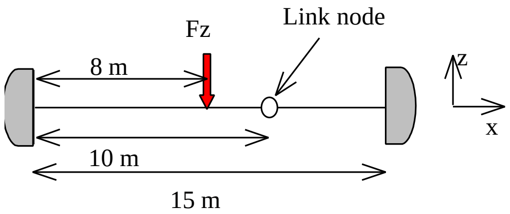
	Beam			Vertical supports				Section					LTB restraints							Loads				Others	
N° Test	1 span	2 spans	Cantilever	Simple supports	Fixed supports	Mixed supports	Cantilever	Uniform section	Non uniform section	Double symmet.	Mono symmet.	T section	Intermediate restraints	Elastic restraints	Point restraint	Continous restraint	At shear centre	At flange level	Special at ends	Point load	Uniform load	At shear centre	AT flange level	Stiffness release in span	
40	X			X					X	X							X			X		X			
41	X			X					X	X							X				X	X			
44	X				X			X		X							X			X		X		X	
50		X		X				X		X							X			X		X			
51		X		X				X		X							X				X	X			
52		X		X				X		X							X				X	X			
53		X		X				X		X							X			X		X			
54		X		X				X		X							X				X	X			
55		X		X				X			X						X			X		X			
56		X		X				X			X						X				X	X			
57		X		X				X			X						X			X		X			
58		X		X				X			X						X				X	X			
60-1			X		X			X		X							X			X			X		
60-2			X		X			X		X							X			X		X			
60-3			X		X			X		X							X			X			X		
61-1			X		X			X		X							X				X		X		
61-2			X		X			X		X							X				X	X			
61-3			X		X			X		X							X				X		X		
62-1			X		X			X		X							X				X		X		
62-2			X		X			X		X							X				X	X			
62-3			X		X			X		X							X				X		X		
65-1			X		X			X			X						X			X			X		
65-2			X		X			X			X						X			X		X			
65-3			X		X			X			X						X			X			X		
66-1			X		X			X			X						X				X		X		
66-2			X		X			X			X						X				X	X			

66-3			X		X			X			X						X				X		X		
67-1			X		X			X			X						X				X		X		
67-2			X		X			X			X						X				X	X			
67-3			X		X			X			X						X				X		X		
70	X			X				X		X							X				X	X			
71	X				X			X		X							X				X	X			
72	X					X		X		X							X				X	X			
75	X			X				X			X						X				X	X			
76	X				X			X			X						X				X	X			
77	X					X		X			X						X				X	X			
80	X			X				X		X			X		X		X				X	X			
82	X			X				X		X			X		X		X				X	X			
83	X			X				X		X			X		X		X				X	X			
84-1	X			X				X		X			X		X			X			X	X			
84-2	X			X				X		X			X		X		X				X	X			
84-3	X			X				X		X			X		X			X			X	X			
85	X			X				X		X			X		X		X				X	X			
86	X			X				X		X			X	X	X		X				X	X			
87	X			X				X		X			X	X	X		X				X	X			
88-1	X			X				X		X			X	X	X			X			X	X			
88-2	X			X				X		X			X	X	X		X				X	X			
88-3	X			X				X		X			X	X	X			X			X	X			
89	X			X				X		X			X	X		X	X				X	X			
90	X			X				X		X			X	X		X	X				X	X			
91	X			X				X		X			X	X		X		X			X	X			
92	X			X				X			X		X	X		X		X			X	X			
100	X			X				X				X					X			X		X			
101	X			X				X				X					X				X	X			
102	X			X				X				X					X			X		X			
103	X			X				X				X					X				X	X			
110-1	X				X			X		X							X				X		X		
110-2	X				X			X		X							X				X	X			
110-3	X				X			X		X							X				X		X		
111-1	X			X				X		X							X		X		X		X		
111-2	X			X				X		X							X		X		X	X			
111-3	X			X				X		X							X		X		X		X		
112-1	X			X				X		X							X		X		X		X		
112-2	X			X				X		X							X		X		X	X			
112-3	X			X				X		X							X		X		X		X		

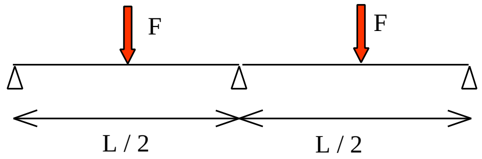
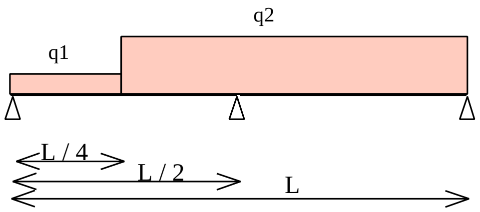
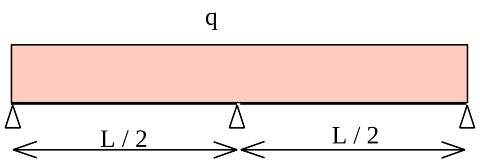
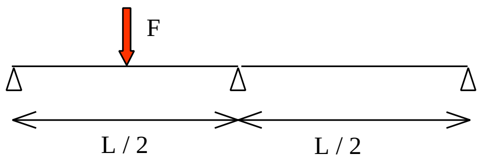
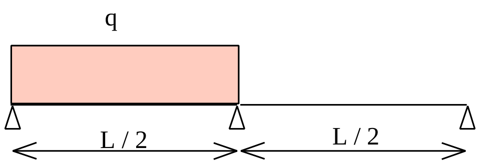
2.3 Results

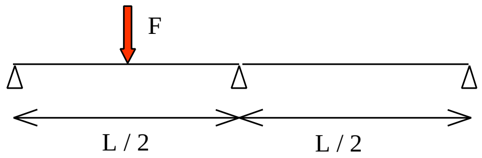
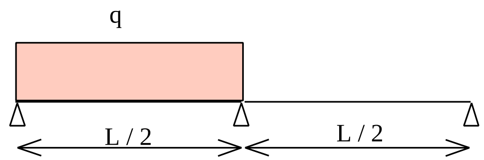
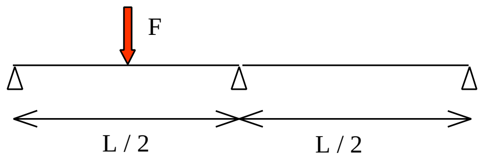
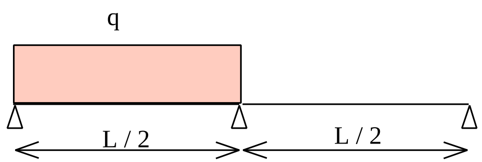
All tests with LTBeam have been performed with N=100 beam elements, except when specified otherwise.

Set 40 : Formatted data

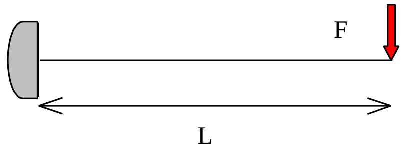
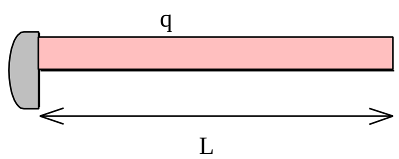
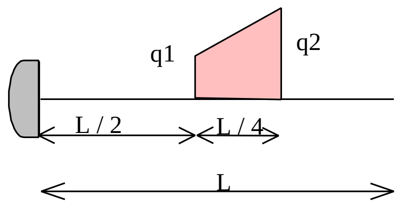
N°	Section	Boundary conditions and loading		Results		
				μ_{LTB}	μ_{ANS}	Δ
40	Double symmetrical I section. Height h varies from 536 mm at the origin to 236 mm at the end of the beam. bf1 = bf2 = 200 mm tf1 = tf2 = 18 mm tw = 12 mm	 N=300	L = 15 m. F = -10 kN. Supports and load are applied at the shear centre line. At each end, v and θ fixed, v' and θ' free.	4,9701	5,0229	0,11 %
41	Idem Test 40	 N=300	L = 15 m. q = -0,2 kN/m Supports and load are applied at the shear centre line. At each end, v and θ fixed, v' and θ' free.	27,588	27,901	0,11 %
44	Double symmetrical I section. h = 310 mm bf1 = bf2 = 190 mm tf1 = tf2 = 13,5 mm tw = 10,5 mm	 Two cantilever beams, connected by a link N=300	DOF transmitted by the link : UX, UY, UZ (v) directly transmitted ; RX (θ) transmitted by a spring (k = 1,5 kNm), RY not transmitted, RZ (v') transmitted by a spring (k = 3,75 kNm). Fz = -10 kN At each end, v, θ, v' and θ' fixed. Supports, link and load are applied at the shear centre line.	9,4421	9,4957	0,56 %

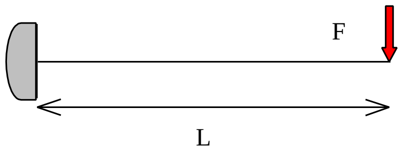
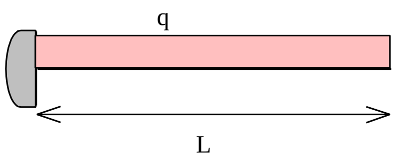
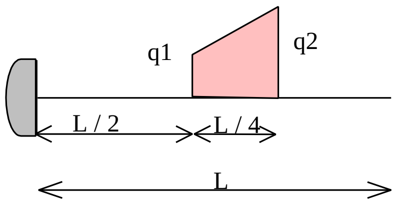
Set 50 : Beams with intermediate support(s)

N°	Section	Boundary conditions and loading	Results		
			μ_{LTB}	μ_{ANS}	Δ
50	Double symmetrical I section. h = 300 mm. bf1 = bf2 = 200 mm tf1 = tf2 = 15 mm tw = 10 mm	 L = 19,5 m. F = -10 kN. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free.	14,966	14,852	0,76 %
51	Idem Test 50	 L = 19,5 m. q1 = -1,0 kN/m and q2 = -3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free.	9,4556	9,4102	0,48 %
52	Idem Test 50	 L = 19,5 m. q = -3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free.	9,48	9,4112	0,73 %
53	Idem Test 50	 L = 19,5 m. F = -10 kN. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free.	12,881	12,810	0,55 %
54	Idem Test 50	 L = 19,5 m. q = -3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free.	7,7618	7,7024	0,77 %

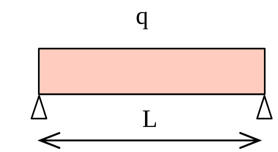
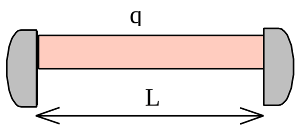
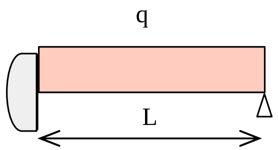
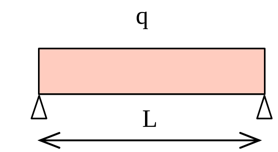
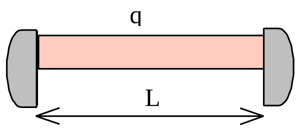
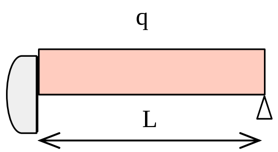
N°	Section	Boundary conditions and loading		Results		
				μ_{LTB}	μ_{ANS}	Δ
55	Mono symmetrical I section. h = 300 mm. bf1 = 150 mm bf2 = 200 mm tf1 = 12 mm tf2 = 15 mm tw = 10 mm		L = 19,5 m. F = -10 kN. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free.	8,1018	8,0646	0,46 %
56	Idem Test 55		L = 19,5 m. q = -3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free.	4,7482	4,7196	0,61 %
57	Mono symmetrical I section. h = 300 mm. bf1 = 200 mm bf2 = 150 mm tf1 = 15 mm tf2 = 12 mm tw = 10 mm		L = 19,5 m. F = -10 kN. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free.	9,2944	9,2530	0,45 %
58	Idem Test 57		L = 19,5 m. q = -3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free.	5,7547	5,7099	0,78 %

Set 60 : Cantilever beams

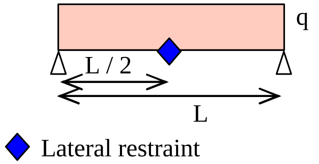
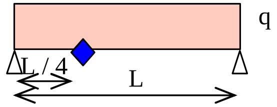
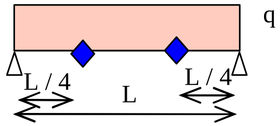
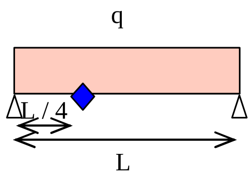
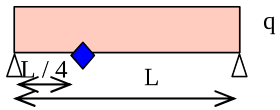
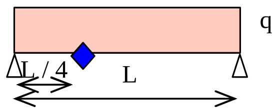
N°	Section	Boundary conditions and loading	Loading location	Results		
				μ_{LTB}	μ_{ANS}	Δ
60-1	Double symmetrical I section. h = 300 mm. bf1 = bf2 = 200 mm tf1 = tf2 = 15 mm tw = 10 mm	 L = 10,0 m. F = -10 kN. At beam fixing, v, θ, v' and θ' fixed.	Upper flange	1,8358	1,8483	0,6 %
60-2			Shear centre	2,3334	2,3434	0,43 %
60-3			Lower flange	2,6707	2,6763	0,21 %
61-1	Idem Test 60	 L = 10,0 m. q = -2 kN/m. At beam fixing, v, θ, v' and θ' fixed.	Upper flange	3,0032	3,0023	0,03 %
61-2			Shear centre	4,238	4,2184	0,5 %
61-3			Lower flange	5,3647	5,3230	0,8 %
62-1	Idem Test 60	 L = 10,0 m. q1 = -2 kN/m, q2 = -5 kN/m At beam fixing, v, θ, v' and θ' fixed.	Upper flange	5,1517	5,1772	0,5 %
62-2			Shear centre	7,5604	7,5509	0,12 %
62-3			Lower flange	9,3689	9,3341	0,37 %

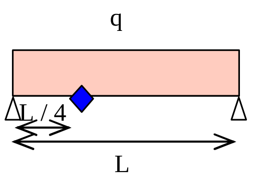
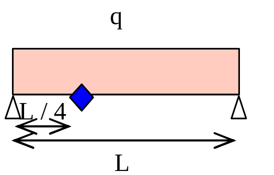
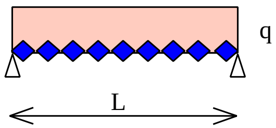
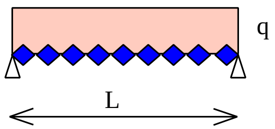
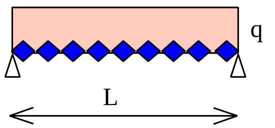
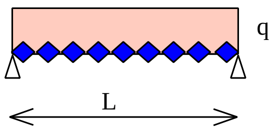
N°	Section	Boundary conditions and loading		Loading location	Results		
					μ_{LTB}	μ_{ANS}	Δ
65-1	Mono symmetrical I section. h = 300 mm. bf1 = 200 mm bf2 = 150 mm tf1 = 15 mm tf2 = 12 mm tw = 10 mm		L = 10,0 m. F = -10 kN. At beam fixing, v, θ, v' and θ' fixed.	Upper flange	1,2065	1,2119	0,47 %
65-2				Shear centre	1,3272	1,3315	0,32 %
65-3				Lower flange	1,5669	1,566	0,06 %
66-1	Idem Test 65		L = 10,0 m. q = -2 kN/m. At beam fixing, v, θ, v' and θ' fixed.	Upper flange	1,9341	1,9307	0,18 %
66-2				Shear centre	2,2432	2,2343	0,40 %
66-3				Lower flange	3,0548	3,0310	0,78 %
67-1	Idem Test 65		L = 10,0 m. q1 = -2 kN/m, q2 = -5 kN/m At beam fixing, v, θ, v' and θ' fixed.	Upper flange	3,3469	3,3530	0,18 %
67-2				Shear center	3,9331	3,9291	0,10 %
67-3				Lower flange	5,1556	5,1277	0,54 %

Set 70 : Beams under constant distributed loading

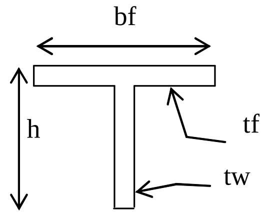
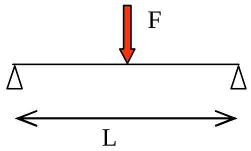
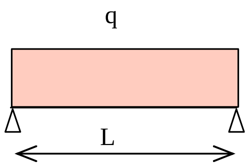
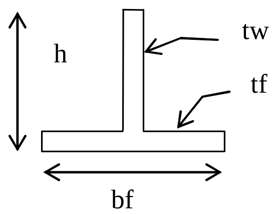
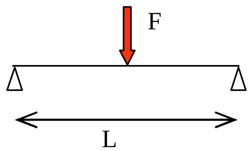
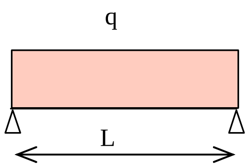
N°	Section	Boundary conditions and loading		Results		
				μ_{LTB}	μ_{ANS}	Δ
70	Double symmetrical I section. h = 350 mm. bf1 = bf2 = 220 mm tf1 = tf2 = 16 mm tw = 11 mm		L = 15 m. q = -2,3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v’ and θ’ free.	2,3077	2,3041	0,15 %
71	Idem Test 70		L = 15 m. q = -2,3 kN/m. Supports and load are applied at the shear centre line. At each end, v, θ, v’ and θ’ fixed.	12,125	12,138	0,11 %
72	Idem Test 70		L = 15 m. q = -2,3 kN/m. Supports and load are applied at the shear centre line. At beam fixing, v, θ, v’ and θ’ fixed. At beam support, v and θ fixed, v’ and θ’ free.	5,5322	5,5215	0,19 %
75	Mono symmetrical I section. h = 350 mm. bf1 = 220 mm bf2 = 120 mm tf1 = 16 mm tf2 = 12 mm tw = 12 mm		L = 15 m. q = -2,3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v’ and θ’ free.	1,6167	1,6102	0,40 %
76	Idem Test 75		L = 15 m. q = -2,3 kN/m. Supports and load are applied at the shear centre line. At each end, v, θ, v’ and θ’ fixed.	6,1237	6,0955	0,46 %
77	Idem Test 75		L = 15 m. q = -2,3 kN/m. Supports and load are applied at the shear centre line. At beam fixing, v, θ, v’ and θ’ fixed. At beam support, v and θ fixed, v’ and θ’ free.	3,4311	3,4171	0,41 %

Set 80 : Beams with intermediate lateral restraint(s).

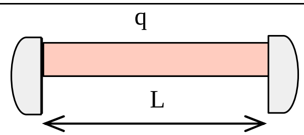
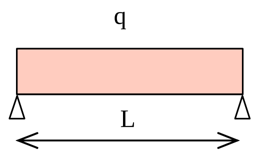
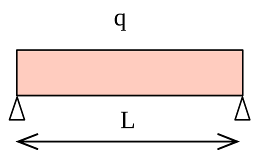
N°	Section	Boundary conditions and loading		Restraint location	Results μ_{LTB}	μ_{ANS}	Δ
80	Double symmetrical I section. h = 500 mm. bf1 = bf2 = 280 mm tf1 = tf2 = 19 mm tw = 12,5 mm		L = 18 m. q = -3 kN/m. Supports, restraint and load are applied at the shear centre line. At each support and at lateral restraint, v and θ fixed, v' and θ' free.		7,0221	6,9718	0,72 %
82	Idem Test 80		L = 18 m. q = -3 kN/m. Supports, restraint and load are applied at the shear centre line. At each support and at lateral restraint, v and θ fixed, v' and θ' free.		5,1471	5,1208	0,51 %
83	Idem Test 80		L = 18 m. q = -3 kN/m. Supports, restraints and load are applied at the shear centre line. At each support and at each lateral restraint, v and θ fixed, v' and θ' free.		10,12	10,041	0,78 %
84-1	Idem Test 80		L = 18 m. q = -3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free. At lateral restraint, v fixed, θ, v' and θ' free.	*Restraint location :* Up flange	5,1471	5,1207	0,51 %
84-2				Shear cent	4,7699	4,7467	0,49 %
84-3				Lo flange	3,0058	2,9876	0,61 %
85	Idem Test 80		L = 18 m. q = -3 kN/m. Supports, restraint and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free. At lateral restraint, θ fixed, v, v' and θ' free.		3,9349	3,9209	0,35 %
86	Idem Test 80		L = 18 m. q = -3 kN/m. Supports, restraint and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free. At lateral restraint, v fixed by a spring (k = 10 kN/m), θ, v' and θ' free.		2,4661	2,4573	0,36 %

N°	Section	Boundary conditions and loading		Restraint location	Results		
					μ_{LTB}	μ_{ANS}	Δ
87	Idem Test 80		L = 18 m. q = -3 kN/m. Supports, restraint and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free. At lateral restraint, θ fixed by a spring (k = 40 kNm), v, v' and θ' free.		2,8378	2,8316	0,22 %
88-1 88-2 88-3	Idem Test 80		L = 18 m. q = -3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free. At lateral restraint, v fixed by a spring (k = 100 kN/m), θ fixed by a spring (k = 50 kNm), v' and θ' free.	*Restraint location :* Up flange Shear cent Lo flange	3,5544 3,3176 3,1143	3,5462 3,3101 3,1072	0,23 % 0,22 % 0,23 %
89	Idem Test 80		L = 18 m. q = -3 kN/m. Supports, restraints and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free. Continuous lateral restraint, v free, θ fixed by continuous spring (k = 1 kNm/m).		2,6682	2,6610	0,27 %
90	Idem Test 80		L = 18 m. q = -3 kN/m. Supports, restraints and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free. Continuous lateral restraint, θ free, v fixed by continuous spring (k = 2 kN/m/m).		2,5868	2,5775	0,36 %
91	Idem Test 80		L = 18 m. q = -3 kN/m. Supports and load are applied at the shear centre line. At each support, v and θ fixed, v' and θ' free. Continuous lateral restraint, θ free, v fixed by continuous spring (k = 0,75 kN/m/m). Continuous restraint applied at upper flange.		2,5453	2,5482	0,11 %
92	Mono symmetrical section. h = 500 mm. bf1 = 280 mm bf2 = 180 mm tf1 = 19 mm tf2 = 15 mm tw = 12,5 mm		Idem Test 91		1,842	1,8389	0,17 %

<u>Set 100 :</u> T Beams

N°	Section	Boundary conditions and loading	Results		
			μ_{LTB}	μ_{ANS}	Δ
100	 h = 150 mm bf = 120 mm tf = 10 mm tw = 8 mm	 L = 10m. F = -5 kN/m. Supports and load are applied on the shear centre line. At each support, v and θ fixed, v' and θ' free.	1,425	1,4266	0,11 %
101	Idem Test 100	 L = 10m. q = -1 kN/m. Supports and load are applied on the shear centre line. At each support, v and θ fixed, v' and θ' free.	1,2044	1,2052	0,07 %
102	 h = 150 mm bf = 120 mm tf = 10 mm tw = 8 mm	 L = 10m. F = -5 kN/m. Supports and load are applied on the shear centre line. At each support, v and θ fixed, v' and θ' free.	1,2564	1,2611	0,37 %
103	Idem Test 102	 L = 10m. q = -1 kN/m. Supports and load are applied on the shear centre line. At each support, v and θ fixed, v' and θ' free.	1,039	1,0429	0,37 %

Set 110 : Boundary restraint conditions

N°	Section	Boundary conditions and loading	Loading location	Results		
				μ_{LTB}	μ_{ANS}	Δ
110	Double symmetrical I section. h = 350 mm. bf1 = bf2 = 220 mm tf1 = tf2 = 16 mm tw = 11 mm	 L = 15 m. q = -2,3 kN/m. At beam fixing, v, θ and v' fixed; θ' free.	Upper flange	5,2142	5,2608	0,88 %
			Shear centre	9,2548	9,2914	0,39 %
			Lower flange	16,212	16,187	0,15 %
111	Idem Test 110	 L = 15 m. q = -2,3 kN/m. Supports are applied at the shear centre line. At each support, v, θ and θ' fixed; v' free.	Upper flange	2,5514	2,5606	0,36 %
			Shear centre	2,8606	2,8706	0,35 %
			Lower flange	3,2065	3,2161	0,30 %
112	Idem Test 110	 L = 15 m. q = -2,3 kN/m. Supports are applied at the shear centre line. At each support, v, θ and v' fixed; θ'free.	Upper flange	3,1042	3,099	0,17 %
			Shear centre	3,9822	3,9821	0,0 %
			Lower flange	5,0317	5,0340	0,05 %

2.4 Conclusions

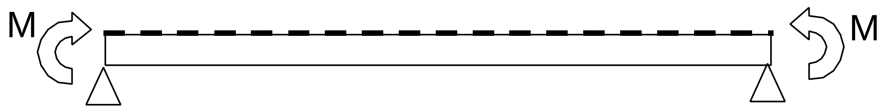
Differences between LTBEAM and ANSYS results are lower than 1% for all tests.

3. COMPARISON TESTS LTBEAM/DRILL

3.1 Presentation

DRILL is a F.E. software (Germany) dedicated to elastic stability of beams. Commercial Intertech (ASTRON Division), partner in the ECSC Project, has got this software and has carried out somme stability analysis of a simply supported beam under uniform bending moment continuously restraint at upper flange by a spring. These results have been compared with LTBEAM results hereafter.

Section : IPE 400 L = 10 m E = 210 000 N/mm² G = 81 000 N/mm²



R_v : continuous restraint stiffness for lateral displacement v [in kN/m/m]
Location at the extreme fibre of the upper flange

R_θ : continuous restraint stiffness for axial rotation θ [in kNm/m]

Δ = LTBEAM/DRILL

3.2 Results

		m_{cr} : multiplicateur critique					
		M = 10 kNm			M = -10 kNm		
R_v	R_q	DRILL	LTBeam	D	DRILL	LTBeam	D
0	0.0	11.86	11.89	0.27%	11.86	11.89	0.27%
0	3.5	15.41	15.44	0.17%	15.41	15.44	0.17%
0	7.0	18.29	18.31	0.09%	18.29	18.31	0.09%
0	10.5	20.77	20.78	0.07%	20.77	20.78	0.07%
500	0.0	79.75	79.89	0.17%	15.17	15.23	0.39%
500	3.5	80.62	80.75	0.16%	23.03	23.09	0.25%
500	7.0	81.47	81.60	0.16%	30.43	30.48	0.15%
500	10.5	82.31	82.44	0.16%	35.96	36.04	0.21%
1000	0.0	102.35	102.48	0.13%	15.38	15.44	0.37%
1000	3.5	103.23	103.36	0.13%	23.68	23.74	0.25%
1000	7.0	104.10	104.23	0.12%	31.69	31.75	0.18%
1000	10.5	104.96	105.09	0.12%	36.58	36.66	0.21%
1500	0.0	124.92	125.05	0.10%	15.45	15.51	0.41%
1500	3.5	125.81	125.95	0.11%	23.93	23.99	0.24%
1500	7.0	126.69	126.83	0.11%	32.19	32.25	0.18%
1500	10.5	127.57	127.70	0.10%	36.91	37.00	0.24%

Note: DRILL : 10 elements ; tolerance 10^{-6}
LTBeam : 100 elements ; tolerance 10^{-6}

3.3 Conclusions

Differences between LTBEAM and DRILL results are lower than 0,5% for all tests.

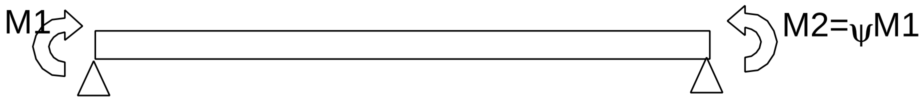
4. COMPARISON WITH TESTS RESULTS FROM LITERATURE

4.1 Presentation

LTBEAM results have been compared with those found in the following reference :

> Braham M. – "*Le déversement élastique des poutres en I à section monosysmétrique soumises à un gradient de moment de flexion*" – Revue Construction Métallique n°1-2001 – CTICM

Tests concern a simply supported beam with equal/unequal end moments M1 and M2 (M2 ≤ M1) with a ratio $\psi = M2/M1$.



Three cross-sections are tested :

- a double symmetrical I cross-section
- a mono-symmetrical I cross section
- a T cross-section

Tests have been carried out with various software and FE codes, and by various authors. **The reader will refer to the above reference for more information about the authors, the programmes used for the numerical simulations, conditions in which these simulations have been performed and cross-section dimensions.**

4.2 Results

4.2.1 Double symmetrical I cross-section

Section properties :

Iw	It	Iz	Iy	A	zs	βz	E	G
cm^6	cm^4	cm^4	cm^4	cm^2	cm	cm	daN/cm^2	daN/cm^2
125930	15.57	602.7	7999	51.89	0	0	2100000	800000

Results (from Table 3 of the reference paper):

		Elastic critical bending Moment left side (M1) in kNm								
Psi = M2/M1	Length in m.	According to Mohri		According to Braham					**CTICM**	
		Abaqus	Mohri	ELTBTB	FinelG	IBDSQ	Abaqus	Ansys	**LTBEAM**	D %
1	3	240	240.43	239.7		240			**239.8**	0.10%
	4	150.53	150.9	149.8		150.1	148.4		**149.8**	0.10%
	5	107.56	107.97	106.9		107.2			**107**	0.38%
	6	83.22	83.03	82.65		82.9	82.3		**82.7**	0.14%
	7	67.8	67.38	67.29		67.5			**67.3**	0.29%
	8	57.23	57.45	56.77		56.97	56.7		**56.8**	0.39%
-1	3	654	657.22	655.5		656			**655.9**	0.03%
	4	408.4	411.46	408.7		409.5	388.2		**408.9**	0.90%
	5	290	293.45	290.9		292			**291**	0.20%
	6	225.4	227.48	224.2		225	220.6		**224.3**	0.11%
	7	183.23	184.53	182		182.6			**182.1**	0.54%
	8	154.26	154.61	153		153.6	152		**153.1**	0.26%

Note : D ***is the difference (absolute value) to the mean value of other available results for the case under consideration.***

4.2.2 Mono-symmetrical I cross-section

Section properties :

Iw	It	Iz	Iy	A	zs	betaz	E	G
cm^6	cm^4	cm^4	cm^4	cm^2	cm	cm	daN/cm^2	daN/cm^2
25081	12.39	335.05	3134.9	43.53	8.8	10.77	2100000	800000

Results (from Table 4 of the reference paper) :

		Elastic critical bending Moment left side (M1) in kNm								
Psi = M2/M1	Length in m.	According to Mohri		According to Braham					**CTICM**	
		Conci	Mohri	ELTBTB	FineIG	IBDSQ	Abaqus	Ansys	**LTBEAM**	D %
1	3	223.8	216.35	220.6	220.1	220.6		223.8	**220.1**	0.35%
	4	125.9	133.18	135.4	135.2	135.4	133.8	137.2	**135.6**	1.40%
	5	94.4	94.086	94.8	94.75	94.9		95.77	**94.9**	0.12%
	6	69.9	71.485	71.9	71.9	72	71.44	72.4	**72**	0.59%
	7	57.5	57.037	57.46	57.5	57.6		57.7	**57.5**	0.06%
	8	49.3	47.377	47.67	47.7	47.8	47.6	47.73	**47.7**	0.38%
0.5	3	285	286.58	290.7	290.1	290.7		294.4	**291.2**	0.56%
	4	174	177.64	178.5	178.3	178.6	176.3	180.8	**178.8**	0.60%
	5	117.5	124.96	125	125	125.1		126.3	**125.1**	0.91%
	6	94.4	94.604	94.8	94.8	94.9	94.2	95.49	**94.9**	0.17%
	7	74.3	75.255	75.7	75.9	75.9		76.1	**75.9**	0.50%
	8	66.1	62.432	62.8	63	63	62.7	62.94	**62.9**	0.60%
0	3	391	393.47	399	398.7	394	388.55	396	**400**	1.42%
	4	243	243.87	246	246	246	242.5	246.6	**246.7**	0.75%
	5	168	172.23	172	173	173	171.1	173.3	**173.2**	0.81%
	6	130	130.82	131	131	131	130.3	131.5	**131.5**	0.53%
	7	104	104.35	105	105	105	104.5	104.9	**105.2**	0.50%
	8	86.5	86.577	87.1	87.3	87.3	86.8	86.89	**87.3**	0.43%
-0.5	3	405	405.57	398.9	404	400		385.5	**399.2**	0.16%
	4	273	270.12	267.5	270.5	268.4	263.2	269.2	**267.7**	0.43%
	5	201	201.71	200.7	202.6	201.3		203.6	**200.8**	0.50%
	6	159.4	160.1	160	161.3	160.6	160	162.2	**160.1**	0.26%
	7	131.9	132.33	132.5	133.3	132.9		133.8	**132.6**	0.14%
	8	112.2	112.13	112.5	113.1	112.8	112.6	113.2	**112.6**	0.04%
-0.75	3		260.91	250.7	254.9	251.8		247.7	**250.8**	0.95%
	4		182.63	175.9	179	176.8		178.8	**176**	1.47%
	5		145.11	137.9	140.2	138.6		140.8	**137.9**	1.87%
	6		120.37	114.9	116.8	115.5		117.1	**114.9**	1.74%
	7		102.83	99.4	101	99.9		100.9	**99.4**	1.39%
	8		90.377	88.05	89.4	88.6		89.03	**88.06**	1.16%
-1	3	174.8	174.84	167.5	170.2	168.4		168.1	**167.6**	1.78%
	4	125.9	124.92	119.5	121.4	120.1	119	122	**119.5**	1.91%
	5	100.7	102.09	94.7	96.3	95.3		96.89	**94.8**	2.93%
	6	85.3	86.27	79.7	81	80.1	80.5	81.18	**79.7**	2.81%
	7	74.3	74.68	69.4	70.5	69.8		70.41	**69.4**	2.96%
	8	66.01	66.17	61.9	62.9	62.3	63	62.51	**61.9**	2.58%

Note : D ***is the difference (absolute value) to the mean value of other available results for the case under consideration.***

4.2.3 T cross-section

Section properties :

Iw	It	Iz	Iy	A	zs	betaz	E	G
cm^6	cm^4	cm^4	cm^4	cm^2	cm	cm	daN/cm^2	daN/cm^2
319.45	9.45	301.77	3134.9	35.83	7.98	11.63	2100000	800000

Results (from Table 5 of the reference paper) :

		Elastic critical bending Moment left side (M1) in kNm								
Psi =	Length	According to Mohri		According to Braham					**CTICM**	
M2/M1	in m.	Conci	Mohri	ELTBTB	FineIG	IBDSQ	Abaqus	Ansys	**LTBEAM**	D %
1	3	208.63	205.38	182.5	182.7	183			**189.6**	1.48%
	4	128.86	124.81	112.7	112.7	113	111.7		**116.4**	0.76%
	5	86.93	87.09	79.2	79.2	79.9			**81.5**	1.17%
	6	62.71	65.64	60.2	60.2	60.5	59.9		**61.7**	0.28%
	7	51.66	52.02	48.2	48.2	48.4			**49.3**	0.80%
	8	43.46	42.73	40	40	40.2	39.9		**40.8**	0.60%
0.5	3	272.38	271.01	240.2	-	-			**249**	4.67%
	4	162.99	166.44	148.4	-	-	147.1		**153**	2.07%
	5	123.79	115.92	104.3	-	-			**107.4**	6.34%
	6	86.93	87.08	79.3	-	-	78.9		**81.4**	1.99%
	7	66.07	68.81	63.5	-	-			**65**	1.70%
	8	57.37	56.38	52.76	-	-	52.5		**53.8**	1.74%
0	3	359.31	364.5	324	324.1	324	318	326.2	**335**	0.21%
	4	217.32	224.37	202.9	202.9	203	200.5	205.1	**209**	0.47%
	5	168.29	157.45	143.7	143.6	144	142.4	145.3	**147.5**	1.17%
	6	118.23	119.13	109.7	109.6	110	108.8	110.8	**112.3**	0.02%
	7	99.35	94.72	88	87.9	88.3	87.34	88.7	**89.9**	0.79%
	8	78.24	78.03	73.1	73	73.4	72.62	73.57	**74.5**	0.09%
-0.5	3	57.95	79.83	117.5	97.4	95.53			**93.5**	4.30%
	4	60.85	77.59	110.1	90.73	88.34	70		**85.9**	3.58%
	5	65.37	81.47	101.3	86.41	83.69			**81.1**	3.05%
	6	62.71	79.91	92.1	83.05	80.2	68.3		**77.5**	0.27%
	7	62.09	75.85	87.4	79.88	77.23			**74.6**	2.47%
	8	64.33	71.19	81.8	76.45		68		**71.9**	0.63%
-0.75	3		49.58	62.1	-				**58.7**	5.12%
	4		48.16	59.7					**54.4**	0.87%
	5		50.87	57					**51.7**	4.14%
	6		50.06	54.2					**49.6**	4.85%
	7		47.6	52					**47.9**	3.82%
	8		44.73	50.3					**46.5**	2.14%
-1	3	30.14	35.89	41.8	43.9				**42.5**	12.04%
	4	30.42	35.28	40.42	41.3		32.8		**39.6**	9.87%
	5	29.21	35.02	39.46	39.55				**37.7**	5.28%
	6	28.98	34.12	38.35	38.1		32.4		**36.2**	5.26%
	7	26.82	33.27	37.18	36.78				**35**	4.44%
	8	24.34	31.16	35.93	35.51		32.4		**33.8**	6.06%

Note : D ***is the difference (absolute value) to the mean value of other available results for the case under consideration.***

4.3 Conclusions

In all cases, LTBEAM results are quite acceptable. For some cases of T-sections, especially negative ψ values, the difference may appear rather large, but for these cases the dispersion of results from all software is rather wide and it can be observed that some results from other authors are somewhat questionnable; However, LTBEAM results are in the range. Of course, we have no information on the modelling, on the meshing and on the real section properties used for most of the simulations performed by other authors than CTICM.

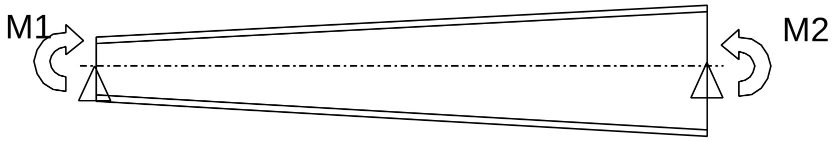
5. TAPERED BEAMS : COMPARISON LTBEAM/FINELG

5.1 Presentation

The results with FINELG software reported herefater have been performed by C. Heck at Liege University in the frame of the Research Contract n°7210 PR 183 with the European Community for Steel and Coal – 1999~2002 (Working Document LTB31).

FINELG is a F.E. code developed by the MSM Department of Liege University.

Tests concern a simply supported I tapered beam under unequal end moments M1 and M2.



The web height varies linearily from hw1 (where M1 is applied) to hw2.

The other section properties are : bf = 250 mm tf = 20 mm tw = 14 mm

$$\Delta = \frac{\left|\mu_{LTBEAM} - \mu_{FINELG}\right|}{\mu_{FINELG}}$$

where μ is the critical load factor.

5.2 Results

N°Test	Span (m)	hw1 (mm)	hw2 (mm)	M1 (kN.m)	M2 (kN.m)	FinelG	LTBeam	D
P1-1A	5	400	800	200	-800	4.482	**4.4885**	0.15%
P1-2A	5	400	800	200	-600	6.352	**6.3617**	0.15%
P1-3A	5	400	800	200	400	5.155	**5.1608**	0.11%
P1-4A	5	400	800	200	600	3.847	**3.8522**	0.14%
P1-5A	5	400	800	200	800	3.062	**3.0661**	0.13%
P1-6A	5	400	800	200	200	7.694	**7.7026**	0.11%
P3-1A	5	200	1000	200	-1200	2.801	**2.8043**	0.12%
P3-4A	5	200	1000	200	1000	2.48	**2.4826**	0.10%
P3-6A	5	200	1000	200	200	7.271	**7.276**	0.07%
P1-1A	10	400	800	200	-800	1.486	**1.4857**	-0.02%
P1-6A	10	400	800	200	200	2.603	**2.6019**	-0.04%
P3-1A	10	200	1000	200	-1200	0.948	**0.9475**	-0.05%

5.3 Conclusion

With all differences lower than 0,2%, the comparison is quite satisfactory.

6. GENERAL CONCLUSION

In the very most of the cases, the difference between LTBEAM results and those from numerical simulations performed with various F.E. software by CTICM or other authors is lower than 1%. For the few cases where this difference is higher, a wide dispersion already exists for the available results from literature and LTBeam results are in the range.

Many other comparisons, not presently reported here, have been made with available results with a very good conclusion
- Tapered beams with unequal end moments – Comparison LTBEAM/FINELG

The general conclusion from the above comparison work is that LTBEAM results are quite satisfactory and that the reliability of LTBEAM seems quite good.

However, users are invited to transmit to CTICM all information about cases for which unsatisfactory results would be found by LTBEAM in order to retrieve the origin of the discrepancy and, if necessary, to correct the programme.

YG